The Art of Computational Science

The Kali Code

vol. 16

Individual Time Steps: Individual Integration Schemes

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Preface

0.1 xxx

We thank xxx, xxx, and xxx for their comments on the manuscript.

Piet Hut and Jun Makino

Chapter 1

XXX

1.1 xxx

In file `world2.rb` many other methods, beside Hermite: forward Euler and leapfrog and most interestingly, a fourth-order multi-step scheme.

Chapter 2

XXX

2.1 xxx

```
def ms4_setup(time)
  @time = @next_time = time
  @acc = @pos*0
  @jerk = @pos*0
  @snap = @pos*0
  @crackle = @pos*0
  @pop = @pos*0
  @nsteps = 0
  @minstep = VERY_LARGE_NUMBER
  @maxstep = 0
end
```

```
def ms4_startup_first_round(acc, jerk, timescale, dt_param, dt_max)
  @acc = acc
  @jerk = jerk
  dt = timescale * dt_param
  dt = dt_max if dt > dt_max
  @next_time = @time + dt
end
```

```
def ms4_startup_second_round(snap, crackle)
  @snap = snap
  @crackle = crackle
end
```

```
def ms4_correct(wa, acc, timescale, dt_param, dt_max)
  d = [[acc]]
  t = [@time]
  dt = [0]
  wa.each do |wp|
    d[0].push(wp.acc)
    d.push([])
    t.push(wp.time)
    dt.push(@time - wp.time)
  end
  for k in (1..wa.size)
    for i in (0..wa.size-k)
      d[k][i] = (d[k-1][i] - d[k-1][i+1])/(t[i]-t[i+k])
    end
  end
  @pop = 24*d[4][0]
  @crackle = 6*(d[3][0] + d[4][0]*(dt[1] + dt[2] + dt[3]))
  @snap = 2*(d[2][0] + d[3][0]*(dt[1]+dt[2]) +
                       d[4][0]*(dt[1]*dt[2] + dt[2]*dt[3] + dt[3]*dt[1]))
  @jerk = d[1][0] + dt[1]*( d[2][0] + dt[2]*( d[3][0] + dt[3]*d[4][0] ) )
  @acc = acc
  dt = wa[0].time - @time
  @vel = wa[0].vel - ( @acc*dt + (1/2.0)*@jerk*dt**2 + (1/6.0)*@snap*dt**3 +
                       (1/24.0)*@crackle*dt**4 + (1/120.0)*@pop*dt**5 )
  @pos = wa[0].pos - ( @vel*dt + (1/2.0)*@acc*dt**2 + (1/6.0)*@jerk*dt**3 +
                       (1/24.0)*@snap*dt**4 + (1/120.0)*@crackle*dt**5 +
                       (1/720.0)*@pop*dt**6 )
  dt = timescale * dt_param
  dt = dt_max if dt > dt_max
  @next_time = @time + dt
  admin(wa[0].time)
  self
end
```

```
def ms4_full_extrapolate(t)
  if t > @next_time
    raise "t = " + t.to_s + " > @next_time = " + @next_time.to_s + "\n"
  end
  wp = WorldPoint.new
  wp.minstep = @minstep
```

```
    wp.maxstep = @maxstep
    wp.nsteps = @nsteps
    wp.mass = @mass
    wp.time = t
    dt = t - @time
    wp.pos = @pos + @vel*dt + (1/2.0)*@acc*dt**2 + (1/6.0)*@jerk*dt**3 +
             (1/24.0)*@snap*dt**4 + (1/120.0)*@crackle*dt**5
    wp.vel = @vel + @acc*dt + (1/2.0)*@jerk*dt**2 + (1/6.0)*@snap*dt**3 +
             (1/24.0)*@crackle*dt**4
    wp.acc = @acc + @jerk*dt + (1/2.0)*@snap*dt**2 + (1/6.0)*@crackle*dt**3
    wp.jerk = @jerk + @snap*dt + (1/2.0)*@crackle*dt**2
    wp.snap = @snap + @crackle*dt
    wp.crackle = @crackle
    wp
  end
```

```
  def ms4_extrapolate(t)
    if t > @next_time
      raise "t = " + t.to_s + " > @next_time = " + @next_time.to_s + "\n"
    end
    wp = WorldPoint.new
    wp.minstep = @minstep
    wp.maxstep = @maxstep
    wp.nsteps = @nsteps
    wp.mass = @mass
    wp.time = t
    dt = t - @time
    wp.pos = @pos + @vel*dt + (1/2.0)*@acc*dt**2 + (1/6.0)*@jerk*dt**3
    wp.vel = @vel + @acc*dt + (1/2.0)*@jerk*dt**2
    wp.acc = @acc + @jerk*dt
    wp.jerk = @jerk
    wp
  end
```

```
  def ms4_interpolate(other, t)
    wp = WorldPoint.new
    wp.minstep = @minstep
    wp.maxstep = @maxstep
    wp.nsteps = @nsteps
    wp.mass = @mass
    wp.time = t
```

```
  dt = other.time - @time
  dt = t - @time
  wp.pos = @pos + @vel*dt + (1/2.0)*@acc*dt**2 + (1/6.0)*@jerk*dt**3 +
           (1/24.0)*@snap*dt**4 + (1/120.0)*@crackle*dt**5 +
           (1/720.0)*@pop*dt**6
  wp.vel = @vel + @acc*dt + (1/2.0)*@jerk*dt**2 + (1/6.0)*@snap*dt**3 +
           (1/24.0)*@crackle*dt**4 + (1/120.0)*@pop*dt**5
  wp
end
```

```
def ms4_setup(time)
  @worldpoint[0].ms4_setup(time)
end
```

```
def ms4_number_of_steps
  4
end
```

```
def ms4_startup_done?(era, dt_max)
  wp = @worldpoint[0]
  if not defined? @startup_cycle
    @startup_cycle = 1
    acc, jerk = era.acc_and_jerk(self, wp)
    timescale = era.timescale(self, wp)
    wp.ms4_startup_first_round(acc, jerk, timescale, @dt_param, dt_max)
    return false
  elsif @startup_cycle == 1
    @startup_cycle += 1
    snap, crackle = era.snap_and_crackle(self, wp)
    wp.ms4_startup_second_round(snap, crackle)
    time = wp.time
    dt = wp.next_time - time
    for i in (1..3)
      @worldpoint.unshift(wp.ms4_full_extrapolate(time - i*dt))
    end
    return true
  else
    raise "@startup_cycle = #{@startup_cycle} != 1"
  end
end
```

```
def take_ms4_step(era, dt_max)
  new_point = ms4_predict
  acc = era.acc(self, new_point)
  timescale = era.timescale(self, new_point)
  k = @worldpoint.size
  wp_array = @worldpoint[k-4...k].reverse
  new_point.ms4_correct(wp_array, acc, timescale, @dt_param, dt_max)
end
```

```
def ms4_predict
  wp = @worldpoint.last
  wp.ms4_extrapolate(wp.next_time)
end
```

nil nil nil nil nil nil nil nil

Chapter 3

Literature References

[to be provided]